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The Topos of Virtuality

Part I: From the Neumann-Nave Topos to the Explication of Conceptual Social Space

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Abstract: *Generalizing the conceptual approach to a theory of biosemiotics which is primarily based on insight from mathematical topology[1], we discuss here the relevance of the cognitive representation of the category of space in terms of the consequences implied by topos theory: In this sense, it is shown that a topos is a Lindenbaum-Tarski algebra for a logical theory whose models are the points of a space. We also show what kind of epistemic conclusions can be drawn from this result with a view to model theory and by doing so establish important relationships among the concepts of social space, networks, systems and evolutionary games on the one hand and semiosis on the other. We can thus achieve a suitable reconciliation of both the onto-epistemic approach of the Kassel group and the evolutionary approach of the Salzburg group, respectively, carrying us forward among other things to fundamental aspects of a unified theory of information. This first paper deals with the mentioned relationships in general spaces, the second[2] deals with applications to virtual space proper.*

Keywords: Topos theory; Topology; Social Space; Cognition; Evolutionary Systems

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[1] Cf. Rainer E. Zimmermann: Topological Aspects of Biosemiotics, tripleC 5(2), 2007, 49-63.
[2] Wolfgang Hofkirchner, Rainer E. Zimmermann: The Topos of Virtuality. Part II. In preparation.

1. Introduction

In his forword to Marcello Barbieri's book[3] Michael Ghiselin points to the process of reconstructing a structure from incomplete information as being one of the most prominent components of *epigenesis* which Barbieri visualizes as the property of a system to increase its own complexity.[4] In fact, Barbieri takes this capacity as a defining property of life itself. And it is accompanied by the properties of attaining both organic memories and organic codes. In fact, as it turns out, this approach is not far from what the Santa Fe school has put forward in the view of defining evolution in terms of an intrinsic unfolding of complexity by systems which tend to optimize their field of possibilities.[5] Indeed, Kauffman actually introduces what he calls a "fourth law" of thermodynamics in order to couple evolution with the practical acquiring of complexity.[6] But in order to phrase concepts like "complexity" and "emergence" in a manner which is sufficiently invariant, it is necessary to develop a new approach by means of a language which is both formal enough so as to cover the logic nucleus of the processes involved (which have their roots in processes usually described within the sciences) and also hermeneutic in the sense that its syntax serves the purpose of illuminating its semantics (as it is usually described in philosophy). A useful and promising approach has been introduced recently by Neumann and Nave which will be the topic of section 2. As it turns out, it is the *language of mathematical categories* that seems appropriate to cover the aforementioned tasks. As one of us has shown at another place[7], a topological view which generically relates to the modern theory of systems is equally promising when trying to phrase the problems involved here.

On the other hand, somewhat earlier, it is Walter Fontana – a Santa Fe protagonist himself – who has shown how to approach an explicit convergence of various theories when describing chemical and biological structures within the framework of what he calls *alchemy*: The idea is that chemical molecules can be visualized as symbolic representations of operators which act upon chemical substances.[8] Insofar there is a structural similarity between a "chemical calculus" and the programming language LISP. This similarity motivates Fontana's "alchemy": An operator $Op = f$ is defined then by its action on relevant variables $x, y, \ldots$: $f(x, y, \ldots)$. The result is the action's evaluation at the location (x, y, …). This viewpoint implies a correspondence table which couples the operator action of molecules to logic:

[3] Marcello Barbieri: The Organic Codes. An Introduction to Semantic Biology. Cambridge University Press, 2003. (The first referee would like to have also mentioned here D. R. Brooks, E. O. Wiley: Evolution of Entropy, University of Chicago Press, 1988, as well as D. Layzer: Cosmogenesis, Harvard University Press, 1990. He argues that these authors were well ahead of Kauffman. We do not challenge this position, but then, obviously, Lee Smolin: The Life of the Cosmos, Oxford University Press, 1997, is certainly ahead of them.)

[4] Ibd., x.

[5] Cf. Stuart Kauffman: Investigations. The Nature of Autonomous Agents and the Worlds They Mutually Create. Oxford University Press, 2000.

[6] The formulation is essentially: Evolution is such that developmental steps from a given state take place in the adjacent possible of this state. The adjacent possible is the set of possible states of a system which has exactly one reaction step distance from that state. Hence, this certifies that the field of possibilities is always larger than the field of actualities, and that evolution is based on local interactions.

[7] Cf. note 3 above.

[8] Walter Fontana: Algorithmic Chemistry. In: C.G.Langton et al. (eds.), Artificial Life II, MIT Press, Boston, 1991.

Physical molecule	Symbolical representation of an operator
Molecule behaviour	Action of the operator
Chemical reaction	Evaluation of a functional term
Chemical properties of binding	Algebraic properties of connectives
Location of the reaction	Proposition
Stable molecule	Cut-free proof of the proposition
Complementarity	Negation
Reaction	Proof with Cut between proposition and negation

We see easily how concepts of the logical calculi of propositions and predicates enter the picture without leaving the framework of chemical reactions. The operator notation can be reproduced straightforwardly by means of the language LISP, because there the standard form of a command is the expression (op x y) where empty input is significant now. Operators can be defined then by the command (define (op name x) (op x y)). And the crucial power of LISP lies in the aspect of self-recursion which admits procedures that can call on themselves. Hence, the natural logic associated with LISP is the Lambda calculus.[9]

Note however that at the same time the same molecular situations represent observable properties of concrete objects that humans can handle in their everyday life. In other words: The concrete structure of observable nature shows up as a pragmatic materialization of a logical language model. Hence, consistent research turns out to be the process of performing self-consistent conventions of logical language models. The actual world (in the cognitive sense) is isomorphic to the world of functions as expressed in terms of operators which are at their base nothing but propositions. Essentially, this constitutes some sort of constructive feedback loop which defines a calculus of objects. The concept of operators serves as the causal connection between the internal structure of an object and the actions by which this same object takes part in the construction of other objects. Hence, there is a *space of possible objects*, and the organization of this space shows up algebraically as a network of mutually mediated paths of production. And here lies the relationship to mathematical topology: This is so because observable forms of objects (shapes) in the usual space of perception are the *phenotypes* of biology. They are points in a space of shapes. Spaces of this latter type can be treated in terms of topology, because they admit consistent criteria of "nearness". In other words: Evolution is determined chiefly by the accessibility of points in shape space.[10] (Note that this is also true – if adequately adapting the terminology – for computer programmes, electronic flow diagrams, urban systems of transport, companies in a given industry and so forth – in short, we talk of percolation problems, and what percolates is always some form of information.)

So after all, the algebraic description is complemented by the topological description, because the nearness of some shape β to another shape α correlates with the probability of a transition from α to β: given the fraction of the boundary which is common to both sets of genotypes of β und α with respect to the total boundary of set α. Hence, $S(\alpha)$ is the set of all (coded) sequences which fold themselves in α. And $\partial S(\alpha)$ is that boundary which can be

[9] This calculus has been utilized recently by Louis Kauffman within the field of theoretical physics, associated with results taken from knot theory. This approach has already established a deep and direct relationship between the fundamental aspects of physics and biology in terms of DNA replication. See for details e.g. Luciano Boi (ed.), Geometries of Nature, Living Systems, and Human Cognition. World Scientific, Singapore etc., 2005.

[10] The first referee prefers to visualize the role of shape space as a constraint on evolution rather than something which chiefly determines the latter. Note however that in this present passage we use the wording in a somewhat generalized sense, because on the one hand, we do refer to general (unspecific) molecular situations, not necessarily to genes only, and on the other hand, we would not like to imply any injective relationships between genotype and phenotype, respectively.

attained by means of one-point mutations of the sequences in $S(\alpha)$. For any two α, β the expression $S(\beta) \cap \partial S(\alpha)$ describes all those sequences which fold in β and are neighbors of those sequences which fold in α. Hence, the *accessibility* of β from α is thus given by $A(\beta \curvearrowleft \alpha) := |S(\beta) \cap \partial S(\alpha)| / |\partial S(\alpha)|$.

Very much on this line of argument will we actually try here to tackle the problems described in the abstract. In the next section 2 we introduce topos theory. In section 3 we utilize the theory for drawing useful interpretations about the spatial nature of topoi. In section 4 we will collect some of the important consequences with a view to social space. Finally, in section 5 we will give a short appendix for displaying some elementary definitions. Of course, we are far from fixed conclusions, let alone from possessing a clear-cut theory. Hence, the approach indicated here is some kind of outline for an ongoing research programme rather than a lecture on established results. But we note that in the meantime, several groups on an international scale have begun to deal with similar ideas and approaches. We are thus, as we feel, on the safe side of a mouvement which is slowly tending towards a conceptual convergence in these matters.

2. The Neuman-Nave Topos

In a recent work of theirs, Neuman and Nave[11] can demonstrate the relevance of (mathematical) categories for cognitively generated concept formation giving concrete examples from child development. Insofar they follow the essential line of argument as given by the late Piaget.[12] The basic idea is to represent the concept construction by means of pushout and pullback diagrams known from category theory:

Take A, B, C as individual cases of some concept D* which is represented by the sign D playing in turn the role of denotating the respective cases as their associated name. Then the *pushout* is defined by a diagram of the following form:

$$\begin{array}{ccccc} D^* & & & & \nwarrow \\ & & \nwarrow & D & \leftarrow & B \\ & \nwarrow & & \uparrow & & \uparrow \\ & & \nwarrow & C & \leftarrow & A \end{array}$$

where upward pointing arrows indicate mappings to D*. In this sense, A is the *domain* of B and C which are in turn the *co-domains* of A. The mappings A → B und A → C are similarity indicators (identifyers) for the cases according to which case A can be consistently classified in order to associate it to a suitable underlying concept. Hence, this relation is a sort of equivalence relation. On the other hand, the mappings B → D and C → D associate the cases with names (they denote them). Then the *pushout* is given by the mapping u: D → D* which associates names with their appropriate concept such that the square diagram as part of the complete diagram commutes with u and the accompanying mappings of the upper left-hand cone. (The *pullback* then is the dual diagram which can be generated by simply reversing all directions of arrows.) The important point is that *only both diagrams together* can mediate case A with the concept D* in question such that it can be properly understood. In other words: The *pushout* refers to the conceptual reconstruction according to the *bottom up method*, while the *pullback* refers instead to the conceptual reconstruction according to the *top down method*.

If we utilize the main example given by Neuman and Nave, we see immediately how to apply pushout and pullback diagrams in practise: Choose the name *Dog* and the associated concept *Dog**. Take as individual cases *C = Chihuahua*, *B = Great Dane*, *A = German Sheperd*. Test then A according to whether it can consistently fall as individual case under the name *Dog.* Obviously, the idea is to look for a similarity criterion in the first place: If A can be shown to be sufficiently similar to B and C, then it falls under D and is thus mediated with the concept D*. The flow of information goes from A to B and C. If both

[11] Yair Neuman, Ophir Nave: A Mathematical Theory of Sign-Mediated Concept Formation. (preprint 2007)

[12] Jean Piaget et al.: Morphisms and Categories. Comparing and Transforming. Terence Brown (ed.), Earlbaum, Hillsdale (N.J.), 1992. – See in particular Gil Henriques: Morphisms and Transformations in the Construction of Invariants. In: id., op.cit., 183-206 (ch. 13).

pushout and pullback exist, we can formulate the result of a deductive algorithm: *If* case B is similar to case A *and if* case C is similar to A, *then* B is similar to C. We notice that the macrolevel (of concepts) and the microlevel (of cases) of reflexion determine each other in a mutual and circular manner. (Hence, the syntactic as well as semantic dynamics of language reproduces the dynamics of self-organizing systems.) Note that propositions of the type *if x* $\cup$ *if y, then z* conform with lines of a computer program. In other words: The representation chosen here illustrates the close relationship between categories and the processing of information (computation).[13]

In fact, Neuman and Nave can show that this dual method of cognition determines the concept formation of humans while in the rest of the animal kingdom the usual method of association and diagonalization turns out to be comparatively uneconomical, because the number of possibilities of conceptual mediations does not only increase exponentially and complexity thus becomes difficult to handle very quickly, but the whole system becomes also very insensitive with respect to a possible change of actual contexts. And more than that: The human way can be additionally extended by means of *metaphorization* such that polysemy and degeneration add to strategic flexibility. Deduction is then replaced by *abductive inference*.

Neuman and Nave give the example of calling a child's aunt A "dog" in metaphorical terms. Then the original diagram takes now the form:

```
D**        ↖
     ↖ D ← B
  ↖    ↑    ↑
     ↖ C ← A
```

where aunt A is being compared with (similar) aunts B and C. However, as the context is shifted now, metaphorization is a mapping of the type D* → D** such that commutation properties stay preserved. This is equally true, if in a more complex case the name D (Dog) is replaced by D' (Hot Dog). Then the diagram must be changed, if applying to dogs in general:

```
D**         ↘
     ↘ D' → B
  ↘    ↓     ↓
     ↘ C → A .
```

Now A means „Dachshound" e.g., and C „Chihuahua". But B is now "sausage". Hence, if D' is Hot Dog, then D* is "sausage*". And instead of utilizing similarity mappings as identifyers ("is like" and "is a" as above), a negation shows up now (is not like) in the diagram, if looking particularly at the mapping D → C. And note the reversal of the arrows directions. Hence, the diagrams represent constraints which act onto the possible interpretations of signs. (And the process of concept formation can thus create dynamical ontologies which are context-depending.[14]) Hence, we have a close relationship between semiosis by means of cognition and communication on the one hand, and logic on the other.[15]

We can generalize now this promising approach by demonstrating in which sense pushout and pullback diagrams as introduced

[13] See also Andreas Blass: Topoi & Computation. (preprint from the website) In this paper the author constructs the exact parallel by utilizing geometric morphisms which correspond to generalized continuous functions.

[14] Note that this does not establish a contradiction to the point of the second referee who visualizes concept formation as context-depending *per se* (and we agree). In this sense, he would also prefer to signify both cognition and communication as conceptual, and we agree again. However, we would not visualize the prime ontological tension between „conceptual" and „real", as he does, but instead between „conceptual" and „modal". This is mainly so, because we start from the essentially Spinozist idea that humans are subjected to what they can observe which is thus nothing but the world *modaliter* (falling into their mode of being). And this is what they actually conceptualize when building theories. While the world *realiter* is not accessible to them in principle. Hence, we would challenge the ontological state of what can be observed. And this has a practical consequence for a meaning which is possibly built into biotic information in the first place. However, the authors of this paper have not yet reached a final conclusion on this latter problem which is prominent in the work of Barbieri as quoted above.

[15] Cf. more recently John C. Baez, Mike Stay (2008): Physics, Topology, Logic and Computation: A Rosetta Stone. From the Baez web page: http://math.ucr.edu/home/baez/ .

by Neuman and Nave into the semiological discussion of concept formation make it possible to define a *topos*.[16] Among other, equivalent definitions, for us important here is the following definition which asserts that *a topos is a category with terminal object and pullbacks, with initial object and pushouts, with exponentials, and with a subobject classifyer.*[17] Note that the first two conditions are clearly demonstrated using the diagrams introduced earlier. Case A of the respective name is an initial object of a category in the first version of the diagrams and a terminal object in the second version.

So what we are talking about here is *a category of denotators* whose objects are the names of type D and whose morphisms are the identifyers (of two types: by denotating of the form "is a" and by metaphorizing of the form "is like"). The various names of individual cases are subobjects of the category. Mappings of the type D → D* and D* → D**, respectively, are functors between categories. (Hence, we differ between the category of denotators and the category of concepts. And we differ between contexts such that the respective category of concepts is different from the one originally associated with the denotators. The first type of functor represents *deduction* (or *induction* as to that) while the second type represents *creative abduction*.)

A *subobject classifyer* is essentially a generalized set of truth values Ω such that the diagram of the form

```
D          ↘
      ↘ A → C
   ↘    ↓     ↓
      ↘ 1 → Ω
```

is a pullback. This time D is the denotator (name), A and C are two individual cases, and the mapping 1 → Ω is a monic "true". (A *monic* is the categorial equivalent of a monomorphism which is an injective homomorphism.) The mapping C → Ω is called *characteristic arrow*. We can visualize what the subobject classifyer is actually doing by thinking of selecting those arrows which "come through" to the "truth" because they imply mutually compatible interpretations of names.[18] Hence, we call the category of denotators utilized in the manner shown above *Neuman-Nave topos* (NN topos). In the following we will have a look at the interpretational consequences of this topos.

3. The Spatial Concept in Topoi

The important point is that a topos turns out to be a Lindenbaum-Tarski algebra for a logical theory whose models are the points of a space.[19] In other words, we can identify an appropriate space with a logical theory such that its points are the models of this theory, its open sets the propositional formulae, the sheafs the predicate formulae, and the continuous maps the transformations of models. At this point logic connects with model theory: Essentially, a Lindenbaum-Tarski algebra A of a logical theory T consists of the equivalence classes of propositions p of the theory under the relation ≅ defined by p ≅ q when p and q are logically equivalent in T.

[16] We follow here the presentations of the topic in various works of standard literature: See e.g. Robert Goldblatt: Topoi. The Categorial Analysis of Logic. North Holland, London, 1984. and with respect to special perspectives chosen when introducing topoi: J. L. Bell: Toposes and Local Set Theories. Clarendon Press, Oxford, 1988. – J. Lambek, P.J.Scott: Introduction to Higher Order Categorical Logic. Cambridge University Press, 1986. Also very important: Saunders MacLane, Ieke Moerdijk: Sheaves in Geometry and Logic: A First Introduction to Topos Theory. Springer, London, 1992. – P. T. Johnstone: Topos Theory. Academic Press, London etc., 1977. – And more recently id.: Sketches of an Elephant: A Topos Theory Compendium. 2 vols. Oxford Science Publications, 2002-2003. – It is noteworthy to fix the terminology chosen by means of the general introduction by Saunders MacLane: Categories for the Working Mathematician. Springer, New York, Berlin, Heidelberg, 1971. as well as with countless generalizing ideas by Peter J. Freyd, Andre Scedrov: Categories, Allegories. North Holland, Amsterdam, 1990.

[17] See also the short summary of details in the appendix to this present paper.

[18] This also clarifies the meaning of the subobjects themselves: Basically, a subobject of a C-object in a category C is thus a monic C-arrow with codomain in the target object. This is so because the domain of a monic is isomorphic to a subset of the codomain. And this also introduces *exponentials* which are simply all morphisms from a domain to a codomain of an object.

[19] We follow here the terminology of Steven Vickers: Locales and toposes as spaces. (preprint, Birmingham, 2004).

That is, in T proposition q can be deduced from p and viceversa. Operations in A are inherited from those available in T, typically conjunction and disjunction. When negation is also present, then A is Boolean, provided the logic is classical. Conversely, for every Boolean algebra A, there is a theory T of classical propositional logic such that the Lindenbaum-Tarski algebra of T is isomorphic to A. In the case of intuitionistic logic, the Lindenbaum-Tarski algebras are Heyting algebras. (Hence, we deal here with an algebra of logical propositions in which logically equivalent formulations of the same proposition are not differentiated.)

We recognize immediately that it is model theory which relates representation with interpretation. (And this is what the diagrams discussed above are all about.) In other words: Model theory is the mathematical discipline that checks semantic elements of structures by means of syntactic elements in a given language. The latter can have logical as well as non-logical symbols and grammatical rules, but in principle, it is always the explication of a logical theory. Is L such a language, and M some set, then M becomes an L-structure by means of the interpretation of each of the non-logical symbols in L. Each proposition which is formulated according to the rules gains some *meaning* in M. Hence, representation entails interpretation and viceversa.

It is not the proper place here to enter deeply into the discussion of model theory.[20] But what we can already notice is the relevance of the spatial approach to topoi: We recall from philosophical epistemology that essentially, a *theory* is a set of propositions which satisfy certain rules. If we visualize the theory as an abstract space, then the points of this space are subsets of propositions. Hence, generalized (abstract) spaces (not only within the field of mathematics) are nothing but sets of propositions or subsets of languages. Obviously, the languages serve the purpose of drafting out a picture of the world so as to orient oneself within its complex network of social and non-social interactions.

This aspect is directly projected onto a plane representing an abstract space of reflexive operations in the case of what we call *glass bead game*.[21] The projection takes place here on a two-dimensional plane which is represented in terms of vertices and edges of a network, where the vertices are points which represent propositions and the edges are logical connectives of these propositions. In principle, this is a graphical representation which maps nicely what the topos concept means when referring to its spatial aspect. The glass bead game consists of sequences of points being consistently connected by appropriate edges such that the resulting path within the network of propositions is the picture of a research process which mirrors the model building common in the sciences. (The idea is taken indeed from the well-known novel of Hermann Hesse's.) Hence, the glass bead game essentially maps a section of social space (namely its scientific section laid down in scientific scripture). And by doing so it illustrates that this space is intrinsically dynamical, because it is actually constituted by the processing of the sequences of propositions according to given rules. In other words: We deal here with the processing of information (including its organization and interpretation). This conception is well compatible with Lorenzer's theory of "language games" stressing the importance of predicators for the explicit training of social interactions in daily life.[22]

One aspect is still missing which is the concrete *multi-perspectivity* of social space. This is in fact dealt with in detail in the work of Mazzola in order to take the various perspectives into account which determine the modes of interpretation of given works of music. But this aspect is equally important for

[20] For a useful survey refer to Wilfrid Hodges: A shorter model theory. Cambridge University Press, 1997.

[21] Cf. Rainer E. Zimmermann: The Modeling of Nature as a Glass Bead Game. In: Eeva Martikainen (ed.), Conference *Human Approaches to the Universe. An Interdisciplinary Perspective*. Helsinki. Agricola Society, 2005, 43-65. More details recently in id.: Was heißt und zu welchem Ende studiert man Design Science? (vol. 1 of Muenchener Schriften zur Design Science), Shaker, Aachen, 2007.

[22] Cf. Alfred Lorenzer: Sprachspiel und Interaktionsformen. Suhrkamp, Frankfurt/M., 1977. as well as id.: Sprachzerstoerung und Rekonstruktion. Suhrkamp, Frankfurt/M., 1970. – Originally, Lorenzer looked for a theoretical combination of Wittgenstein's and Freud's approaches.

social spaces in general. And as it turns out, it can also be included in the terminology of topos theory. This can be shown in terms of what is called “Yoneda lemma”:

For an arbitrary pre-sheaf P in C there is a bijection between natural transformations y(C) → P and elements of the set P(C) of the form:

$$\theta: \mathrm{Hom}_{\mathcal{C}}\,(y(C), P) \rightarrow^{\approx} P(C).$$

Here $\mathcal{C}$ is the category of all pre-sheaves of C, where C is a fixed small category and C^{opp} is its opposite. The objects of $\mathcal{C}$ are the functors C^{opp} → Sets, and the arrows (morphisms) are all natural transformations between them. Each object c of C gives rise to a pre-sheaf y(c) on C defined on an object D of C by $y(C)(D) := \mathrm{Hom}_{\mathcal{C}}\,(D, C)$ and on a morphism α: D’ → D by $y(C)(\alpha)$: $\mathrm{Hom}_{\mathcal{C}}\,(D, C) \rightarrow \mathrm{Hom}_{\mathcal{C}}\,(D', C)$. Pre-sheaves of this form are called *representable*, and in this case y is called an *Yoneda embedding* which is a special case of the lemma quoted above.

For Mazzola, what the Yoneda lemma clarifies, is that it serves as a foundation of multi-perspectivity among local interpretations: In music, let R and S be appropriate vector spaces, and let K in R and L in S be two local compositions.[23] The relations then between the two compositions can be expressed as a morphism K → L. Essentially, this morphism defines a perspective under which L can be seen. (In fact, we can construct similar pushout and pullback diagrams as shown in the case of the NN topos.) The Yoneda lemma certifies then that the system of all L-perspectives determines the isomorphy class of L. In other words: The morphisms can be visualized as essentially hermeneutic instruments in order to classify and understand local compositions. It is quite straightforward then to generalize this aspect to more “unspecialized” cases as instances of social space. The important point is that most of the time we do not talk here about a space *as it is actually observed*, but instead about a space *as it could be observed*. In other words: The number of possible interpretations is larger than the number of actual interpretations. (Remember that in common social space collections of these interpretations form the practical “world-view”.) Hence, not only does space show up as social space in the first place, and not only does social space show up as a space whose points are propositions of logical theories, but moreover social space shows up as well as a *virtual space*. Strictly speaking then, social space is a *special case* of virtual space, and not viceversa, because the latter’s “virtuality” refers to the field of possibilities rather than to the field of actualities which can be empirically observed.[24]

4. Conclusions

What we see now is that traditionally, there have been already many connections between the human techniques of spatial representation (what has been called *anthropological graphism* elsewhere[25]) and the mapping of processes in terms of logical formulae. The approach of Fontana is one example, very much on the line of the Santa

[23] We refer here to an earlier paper of Mazzola’s: Topologien gestalteter Motive in Kompositionen. (reprint from the website, 1997) The complete outline of Mazzola’s approach is given in the monumental book id.: The Topos of Music. Geometric Logic of Concepts, Theory, and Performance. Birkhaeuser, Basel, Boston, Berlin, 2002.

[24] As the second referee correctly states, virtual space is an extremely rich and complex concept. But note that, in principle, it is identical with what we might call *reality*, while social space, in so far it is observable, is *modality* instead. In fact, as far as it goes, communication in terms of language is concrete rather than abstract, though ist interpretations are abstract rather than concrete, but can unfold concrete actions undertaken. Hence, the second referee’s objections as to the fact that this paper, by having been shown how humans construct spatial representations by editing propositions of theories, covers only half the story of social space, are still topical in the ongoing discussions. The authors thank this referee for the stimulating input.

[25] Rainer E. Zimmermann: Graphismus & Repräsentation. Zu einer poetischen Logik von Raum und Zeit. Magenta, München, 2004. – The idea goes back to a formulation of Henri Lefebvre: The Production of Space, Blackwell, Oxford, 1991 (1974), 33: “A conceptual triad has now emerged …: 1) *spatial practice* which embraces production and reproduction, and the particular locations and spatial sets characteristic of each social formation. … 2) *representations of space* which are tied to the relations of production …, hence to knowledge, signs, codes … 3) *representational spaces* embodying complex symbolisms … linked to the clandestine side of social life …” Note that in this book the problem of space is posed for the first time in a sufficiently modern language. There are even a remarks on Hesse’s glass bead game (ibid., 24, 136).

Fe school on self-organized criticality. We have also seen that this kind of discussion visualizes processes in the general sense as percolation phenomena[26], and what is being percolated is information then. And we have seen that it is topos theory that provides an appropriate language in order to deal with these aspects of spatial representation. More than that: A topos can be essentially interpreted as the algebraic expression of the fact that spaces utilized in human cognition are basically constituted by propositions of logical theories. On the other hand, the procedures of deduction and induction as well as creative abduction, available to human logic, can be rephrased in terms of algorithmic procedures. Hence, they are both accessible by means of programmes as they are utilized in computation, and by means of game theory, because on a fundamental level of reflexion games are essentially algorithmic procedures whose strategies are given by its rules.[27] What we realize then is that all of this relates nicely with the approaches of the Kassel and Salzburg schools as described at earlier occasions.[28]

Note that the conceptual nucleus of these approaches is given by two triadic arrangements of concepts of the form:

Cognition	Communication	Co-operation
Space	Network	System

The first triadic structure mirrors the close relationship between cognition and communication on the one hand – as pair of concepts characterizing the process of reflexion – and co-operation on the other hand – as characterizing the transition from reflexion to action.[29] While the first pair of concepts cannot be separated in practise, the latter concept is structurally separable from the other two. Reflexion and action represent thus two different time scales which show up with the systematic updating process involved in the sequential organization which is underlying both reflexion and action, respectively. The producing of models belongs to the pair of concepts in the first place and is primarily based on a generic self-model which defines the framework according to which cognition is normalized. Essentially, this is the onto-epistemic picture of the grasping of the world by humans.[30] Earlier stages of evolution can be visualized as conceptual approximations of this onto-epistemic picture. In *methodological* terms the second triadic structure is associated with the first such that there are intrinsic pairwise correspondences between cognition and space, communication and network, and co-operation and system, respectively. In other words: Space is the conceptual structure from which that world of daily life is being reconstructed which is derived from the process of cognition. Network is the

[26] Cf. Dietrich Stauffer, Amnon Aharony: Introduction to Percolation Theory. Taylor & Francis, London, 2nd ed., 1994.

[27] Cf. Robin Houston: Categories of Games. Master thesis, University of Manchester, 2003.

[28] Cf. the volumes of collected essays presenting the results of the INTAS co-operation project "Human Systems in Transition" with the universities of Vienna, Kassel, Kyiv, and the Academy of Sciences, Moscow led by Wolfgang Hofkirchner (then Vienna, now Salzburg), namely by V. Arshinov, C. Fuchs (eds.): Causality, Emergence, Self-Organization (Volume 1), Russian Academy of Science, NIA-Priroda, Moscow, 2003. Also I. Dobronravova, W. Hofkirchner (eds.): Science of Self-Organization and Self-Organization of Science (Volume 2), Abris, Kyiv, 2004. And R. E. Zimmermann, V. Budanov (eds.): Towards Otherland. Languages of Science and Languages Beyond. Kassel University Press, 2005.

[29] See for more details Rainer E. Zimmermann: Konzeptuelle Dialektik (Conference of the Ernst Bloch association on Polyphonic Dialectics, Berlin, 2007), in: Doris Zeilinger (ed.), VorSchein, Nuernberg, in press. (2008)

[30] In other words: It is the human mode of being to produce knowledge. Hence, for humans, ontology and epistemology fall into one. Higher and lower animals, in principle also plants, represent the same scheme, but on lower levels of organization. Essentially, even physical systems on a very fundamental level can be thought of as satisfying the general framework of this scheme, though by extremely simple means of organization. In the sense of Stuart Kauffman, the most fundamental physical (autonomous) agent can be defined by satisfying a minimal condition from thermodynamics: namely that the system is able to perform at least one thermodynamic work cycle. This is probably true for spin networks on the level of quantized physical space. Hence, evolution shows up as a multi-shifted hierarchy of complexity as to the unfolding of various forms of organized collectives of (autonomous) agents. Humans represent thus systems with (up to now) maximal degree of organization. In between we would expect a manifold of biological structures with different degrees of organization smaller than that degree in humans.

conceptual structure from which those social interactions of daily life can be reconstructed which are derived from the process of communication. System is the conceptual structure from which those joint manipulations of the material world can be reconstructed which are derived from the process of co-operation. Obviously, the first and second pair of concepts from the two triadic structures regulate the actual flow of information and the interpretation of meaning while the third pair regulates the production of matter. This is a result of the fact that the complete system is more than space and network, because it does not only encompass social interactions, but also tangible matter.[31] In a sense, space is the region in which the system unfolds its actions, while the network is skeleton of both space and system. Hence, this present paper can show clearly how humans construct their various spatial representations by means of editing the propositions of their theories.

5. Appendix: Elementary Details on Categories

We shortly summarize now some elementary definitions and properties concerning mathematical categories[32]: A *category* C consists of 1) a collection of objects Ob(C) and 2) a set hom(x, y) of *morphisms* for each pair of objects x, y from x to y equipped with a) an identity morphism of the form $1_x : x \to x$ and b) a morphism $f \circ g: x \to z$ for each pair of morphisms $f: x \to y$, $g: y \to z$ called *composition* such that (i) for each morphism f the left and right laws of identity are valid: $1_x \circ f = f = f \circ 1_y$ and (ii) for each triple of morphisms the law of associativity is valid: $(f \circ g) \circ h = f \circ (g \circ h)$. In particular, an *isomorphism* is a morphism which has an inverse. Given categories C and D, a *functor* $F: C \to D$ consists then of 1) a function $F: Ob(C) \to Ob(D)$ and 2) a function $F: hom(x, y) \to hom(F(x), F(y))$ for each pair $(x, y) \in Ob(C)$ such that a) F preserves identities: that is, for each object $x \in Ob(C)$, $F(1_x) = 1_{F(x)}$, and b) F preserves compositions: that is, for each pair of morphisms f, g in C we have: $F(f \circ g) = F(f) \circ F(g)$. Given two functors $F, G: C \to D$, then a *natural transformation* $\alpha: F \Rightarrow G$ consists of a function α which maps each object $x \in C$ to a morphism $\alpha_x: F(x) \to G(x)$ such that for each morphism $f: x \to y$ in C the following diagram commutes:

$$\begin{array}{ccc} F(x) & \xrightarrow{F(f)} & F(y) \\ \downarrow\alpha_x & & \downarrow\alpha_y \\ G(x) & \xrightarrow{G(f)} & G(y)\ . \end{array}$$

It can be shown straightforwardly that identities, compositions and the law of associativity are being preserved for natural transformations. Given two functors, a *natural isomorphism* is a natural transformation which has an inverse. Insofar a natural transformation is a natural isomorphism iff (if and only if) for each object $x \in C$ the morphism α_x is invertible. A functor $F: C \to D$ is an *equivalence*, if it has a weak inverse, i.e. a functor $G: D \to C$ such that there are natural isomorphisms $\alpha: FG \Rightarrow 1_C$ and $\beta: GF \Rightarrow 1_D$.

A *monoidal category* (or monoid) consists of 1) a category M, 2) a functor called *tensor product*, of the form $\otimes: M \times M \to M$ with $\otimes(x, y) = x \otimes y$ and $\otimes(f, g) = f \otimes g$ for objects $x, y \in M$ and morphisms f, g in M, 3) an identity object $1 \in M$, 4) natural isomorphisms called *associators*: $a_{x,y,z}: (x \otimes y) \otimes z \to x \otimes (y \otimes z)$ satisfying the left and right lwas of identity: $l_x: 1 \otimes x \to x$, $r_x: x \otimes 1 \to x$, such that a) the following diagram commutes for all objects $w, x, y, z \in M$ (Pentagon equation):

$$\begin{array}{ccccc} & & (w \otimes x) \otimes (y \otimes z) & & \\ & \nearrow & & \searrow & \\ ((w \otimes x) \otimes y) \otimes z & & & & w \otimes (x \otimes (y \otimes z)) \\ & \searrow & & \nearrow & \\ & (w \otimes (x \otimes y)) \otimes z & \to & w \otimes ((x \otimes y) \otimes z) & \end{array}$$

[31] Of course, these two make no difference with respect to both the energy balance and the entropy balance. Matter belongs to the additional term which has to be added on the entropy's side in order to make both balances equal, because it can be visualized as a kind of stored information (memory).

[32] We follow here the terminology and convention of John Baez put forward on his website when discussing categories [utilize the search function on this site]: http://math.ucr.edu/home/baez/.

and b) the triangle equations are valid, i.e. the following diagram commutes:

$$(x \otimes 1) \otimes y \rightarrow x \otimes (1 \otimes y)$$
$$\searrow \quad \swarrow$$
$$x \otimes y .$$

(We skipped here the signifying of the respective associators on their arrows.)

Important is the *principle of duality*: This means essentially the inversion of the directions of all arrows. Hence, for propositions of the type f: $a \rightarrow b$, a = dom f, h = g o f the duals are the propositions f: $b \rightarrow a$, a = cod f, h = f o g. The principle states then that if the proposition Σ is a consequence of given axioms, then so is the dual proposition Σ^*. Dual categories will be signified by an upper index „opp" (opposite).

If for two categories C, D the respective hom(x, y) are sets, then we say that if there are also two functors F: $C \rightarrow D$ and G: $D \rightarrow C$, category C is *left-adjoint* to D (or equivalently: G is *right-adjoint* to F (F⊣G), if the functors Hom_D (F): C^{opp} x D $\rightarrow$ Sets and Hom_C (G): C^{opp} x D $\rightarrow$ Sets are mutually isomorphic. A *terminal object* 1 in a category C is an object which admits exactly one morphism from each object x of the form !: $x \rightarrow 1$. An *initial object* 0 is a terminal object in the category C^{opp}. A *quiver* is a pair G = (A, V), where the elements of V are called *vertices* and the elements of A *arrows*. If each pair of vertices is at most top and bottom of one arrow, then the quiver is called *directed graph*. Obviously, the category of paths P(G) has the paths themselves as its morphisms.

Given a quiver and a category C, and also a diagram Δ from P(C). Given then an object c in C, there is the constant diagram [c], which associates with each vertex in C this very c and with each arrow the identity id_c. A natural transformation $[c] \rightarrow \Delta$ is called *cone on* Δ, written as K(Δ), while a natural transformation $\Delta \rightarrow [c]$ is called *co-cone on* Δ, written as KK(Δ). (In a cone all arrows which start from c have to commute with the arrows of the diagram. In a co-cone all arrows which end in c have to commute with the arrows of the diagram.) Then a *limit* of Δ is a terminal object in K(Δ). While a *co-limit* of Δ is an initial object in KK(Δ). If the diagram is a pair of the form f: $a \rightarrow c$, g: $b \rightarrow c$, then the limit is called *fibre product* or *pullback* of f, g. If the diagram is a pair of the form f: $c \rightarrow a$, g: $c \rightarrow b$, then the co-limit is called *fibre sum* or *pushout* of f, g. In particular, a category is called *finitely (co-) complete* iff it has (co-) limits for all finite diagrams.

For any category C the following propositions are equivalent: 1) C is finitely complete. 2) C has finite products and equalizers. (These are limits of pairs of arrows.) 3) C has a terminal object and pullbacks. Hence, the dual is also valid: 1) C is finitely co-complete. 2) C has finite sums and co-equalizers. 3) C has an initial object and pushouts. It is here where the subobject classifyer comes into play which serves as a categorial analogue for the characteristic functions of set theory. The direct way of defining this leads via sheaf theory:

Be M a partially ordered set: A function which associates with each $p \in M$ a set X_p and with each pair $p \leq q$ a mapping X_{qp}: $X_q \rightarrow X_p$ such that X_{pp} = id (X_p), and, whenever $p \leq q \leq r$, $X_{rp} = X_{qp}$ o X_{rq}, is called *pre-sheaf X on M.* A *subobject* K then of the pre-sheaf X is essentially another pre-sheaf with a similar mapping, K_{qp} say, which is a restriction of X_{qp}. The collection of all pre-sheaves on a partially ordered set M is itself a category called Set^M. (This can be shown to be a topos, because pre-sheaves can be alternatively defined in terms of *sieves* which are nothing but collections of morphisms acting on objects of M such that compositions are being preserved. The important property of sieves is that they imply the existence of subobject classifyers which have the structure of Heyting algebras.) We can also formulate that the category of functors $C^{opp} \rightarrow$ Sets of a given category C is the category of pre-sheaves of C. Given then for some category C a morphism f: $H \rightarrow G$ in the category of pre-sheaves of C. Then we have: 1) It is f a monomorphism iff Af: $AH \rightarrow AG$ is injective for all objects A of C (the products meaning here "evaluation at A"). 2) It is f an epimorphism, if Af is surjective. 3) It is f an isomorphism, if Af is bijective, respectively. Then we have the following important definition:

Given a complete category C which has a terminal object 1, then a monomorphism *true*: $1 \to \Omega$ in C is called *subobject classifyer* iff, given some monomorphism σ: $S \to X$ in C, there is a unique morphism τ: $X \to \Omega$ such that the diagram

$$\begin{array}{ccc} S & \xrightarrow{\sigma} & X \\ \downarrow ! & & \downarrow \tau \\ 1 & \xrightarrow{true} & \Omega \end{array}$$

is a pullback. (Subobject classifyers are unique up to isomorphisms.) In this sense a subobject of X is an equivalence class of monomorphisms of type σ. If all subobjects of X represent a set for each object X in C, then this set is a pre-sheaf of C. A category is called *cartesian closed* iff it has finite products and each of its elements is exponentiable, i.e. if the functor AX: Sets $\to$ Sets: $X \to A \times X$ has a right-adjoint. (This is particularly true for the category of sets called *Sets*. Then the respective functor is X^A: Sets $\to$ Sets: $X \to X^A$. And the "power set" represents the functor $X \to$ hom(A x X, X).)

For a category C the following groups of properties are equivalent: 1) C is cartesian closed and has a subobject classifyer. 2) C is cartesian closed, finitely co-complete, and has a subobject classifyer. 3) C has a terminal object and pullbacks, exponentials, and a subobject classifyer. 4) C has a terminal object and pullbacks, an initial object and pushouts, exponentials, and a subobject classifyer. 5) C is finitely complete and has power objects.

A category which has these equivalent groups of properties is called (elementary) topos. (In particular it can be shown that the category of pre-sheaves for some category C is a topos.) Functors between topoi that preserve limits, exponentials, and subobject classifyers are called *logical morphisms*.

Take a *site* which a pair (C, J) with J a Grothendieck topology on C (in which sheaves take the role of open sets in terms of classical topology), then a pre-sheaf P in C is a *sheaf* iff for each sieve of J the canonical diagram is an equalizer. The category then which is equivalent to the category of sheaves Sh(C, J) is called *Grothendieck topos*. Note that a Grothendieck topos is an elementary topos.

This is where logic enters again: We go back to the case of orderings. Then a *pre-order* is a binary equivalence relation which is reflexive as well as transitive. It is called *partial order*, if it is also anti-symmetric (pRq, qRp $\Rightarrow$ p = q). Instead of pRq we can alternatively write $p \subseteq q$ such that a power set can be expressed by P = (P, $\subseteq$). We call a power set which can be defined in terms of partial order a *lattice*. For any $x,y \in P$ the products $x \cap y$ and $x \cup y$ are called *meet* and *join*, respectively. The first case is meant to be the greatest lower bound, the second the least upper bound of x and y in the sense of a subset filtering. A lattice is called *bounded*, if there is some y complementary to x such that $x \cup y = 1 \wedge x \cap y = 0$. A bounded lattice is called *complemented*, if for each of its elements there is a complementary one within the lattice. A lattice is called *distributive*, if for each element there is at most one complementary element. In fact, the operations $\cap$ (meet), $\cup$ (join) and „complementarization" represent in the case of the set of truth values 2 the operations conjunction, disjunction, and negation (including implication). Hence, we can rewrite algebraic expressions as logic expressions such that diagrams commute; e.g. for the case of negation we have:

$$\begin{array}{ccc} 1 & \xrightarrow{false} & 2 \\ \downarrow & & \downarrow \\ 1 & \xrightarrow{true} & 2 \end{array}$$

When passing over from Boolean to intuitionistic logic, then we have to take into account that the former is governed by Boolean algebras and the latter by Heyting algebras. The important difference between these two is in the negation operation. In the Boolean case negation shows up as "complementarization", similar to the case of duality: (x) = x. In other words: The negation of the negation reproduces the proposition again. In the case of a Heyting algebra however, this is not true anymore. Instead, we have now: (x) $\neq$ x. This means in particular that recursive operations make the emergence of innovative structures possible. Categories become time-dependent, contrary to sets, because objects can be created as well as annihilated. (Already in the calculus of propisitons can we find the consequences of this formal difference between the types of algebra.[33]) Basically, the calculus of propositions deals with the evaluation ε of propositions of a given set (as mapping from this set onto a lattice A) in terms of their validity: A proposition α in this sense is *A-valid* iff $\varepsilon(\alpha) = 1$ („true“) under given rules. In the case of predicate logic the results of propositional calculus are being preserved (under appropriate generalizations and by introducing quantizing operators). We find that if f: A $\rightarrow$ B is a morphism in the topos C, then the functor Sub(B) $\rightarrow$ Sub(A) of Heyting algebras has a right-adjoint $\forall$ as well as a left-adjoint $\exists$. (Obviously, Heyting algebras and thus intuitionistic logic is more generic for topos theory than Boolean classical logic.)

There is an interesting hermeneutic side-aspect to all of this: In the Lacanian terminology of negations the classical Aristotelian logic is being generalized in terms of a philosophical rather than mathematical discourse. For the classical tradition there is a total (absolute) truth pointing to the complete conformity of language and being which transcends the merely partial truth implied by the analytic discourse. For Aristotle, universality implies existence.[34] For Lacan however there is an implicit „schismogenesis“ of writing and talking such that a generalization is necessary which takes this difference into account. For him, this results in an extension of the negation operations as indicated by the following schematic table:

$\forall$x. f(x) $\exists$x. f(x)

$\forall$x. f(x)	$\exists$x. f(x)
$\forall$x. f(x)	$\exists$x. f(x)
$\forall$x. f(x)	$\exists$x. f(x)

Nothing comes to its determination except by means of difference (as Spinoza already knew), and the universal is not the universal of an essence. In fact, the potential of denotation is questioned in the process of signification. Lacan's *ré-écriture* constitutes thus a new sort of scripture which represents/misrepresents the limits of the scripture of science. In a sense, Lacan generalizes the idea of Goedel's by pointing to a beyond of scientific scripture. Hence, he introduces two qualified negations which replace the classical negation and add two new propositional forms to the table above:

$\forall$ discordant negation
(denying/disclaiming negation)
$\exists$ rejecting negation .

Hence, the field of science's scripture is the world, but the truth of significants cannot be formulated under the form of knowledge. Knowledge means to signify what is part of the world. The sort of knowledge therefore which is expressed within the psycho-analytic discourse is instead of the type of the *mathema* which is the generalized form of the table above. For us here, this aspect is important with a view to the recent enterprises in theoretical physics as well as in music theory (and somewhere in between) to actually introduce a formal language which is capable of achieving a unified discourse with a logical nucleus and a hermeneutic halo for both syntax and semantics.[35]

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[33] We have discussed the more elementary aspects of logic, especially with a view to the classical form of propositional calculus, e.g. with respect to the *modus ponens* elsewhere. See e.g. in: Werner Loh, Ram A. Mall, Rainer E. Zimmermann: Interkulturelle Logik, Mentis, Paderborn, in print (2008), part 3.

[34] Alain Juranville: Lacan und die Philosophie. Boer, München, 1990 (PUF 1984), 397.

[35] We refer here to the cases mentioned earlier of ongoing research by Christopher Isham (London) et al. on the one hand and Guerino Mazzola et al. (Zuerich) on the other.

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